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Quasi Dα- Normal Spaces, πGDα-Closed Sets and Some Functions

Abstract

In aim this paper, we introduce a new concept of quasi-normal spaces called quasi Dα-normal spaces and obtain characterizations and preservation theorems of quasi Dα-normal. The notion can be applied for investigating many other properties.

Keywords: Dα-closed, Dαg-closed πgDα-closed, Dα-open Dαg-open,πgDα-open sets, πgDα-closed, almost πgDα-closed, πgDα-continuous and almost πgDα-continuous functions, Dα-normal spaces, mildly Dα-normal spaces and quasi Dα-normal spaces.

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Introduction

In this paper, we introduce the notion of Dαg-closed, Dαg-open, πgDα-closed,πgDα-open sets,πgDα-closed, almost πgDα-closed, πgDα-continuous and almost πgDα-continuous functions and its properties are studied. Further we introduce a new concept of quasi-normal spaces called quasi Dα-normal spaces and obtain characterizations and preservation theorems of quasi Dα-normal.

Aim of the Study

In aim this paper, we introduce a new class of sets called Dαg-closed, πgDα-closed sets and its properties are studied and we introduce a new concept of quasi-normal spaces called quasi Dα-normal spaces by using Dα-open sets due to Sayed and Khalil[11] in topological spaces and obtained several characterization and preservation theorems for quasi Dα-normal spaces. We insure the existence of utility for new results using separation axioms in topological spaces which is separate on a known separation axioms in topological spaces.

Review of Literature

The notion of quasi normal space was introduced by Zaitsev. [13] Dontchev and Noiri[2] introduce the notion of πg-closed sets as a weak form of g-closed sets due to Levine [**6**]. By using πg-closed sets, Dontchev and Noiri [**2**] obtained a new characterization of quasi normal spaces. Sayed and Khalil [**11**] introduced the concept of Dα-closed sets and discuss some of their basic properties. Recently, Reena et al. [**8**] introduced the concepts of quasi b^+-normal spaces in topological spaces by using b^+ open sets in topological spaces and obtained some characterizations of such spaces.

Preliminaries

Definition

A subset A of a topological space X is called.

Regular closed [12]) If A = Cl(Int(A)).

Generalized closed [4] (Briefly, **g-closed**) if Cl(A) $\subset$ U whenever A $\subset$ U and U is open in X .

πg-closed [2] If Cl(A) $\subset$ U whenever A $\subset$ U and U is π-open in X.

α-closed [7] If Cl(Int(Cl(A))) $\subseteq$ A .

αg-closed [5] If α-Cl(A) $\subseteq$ U whenever A $\subseteq$ U and U is in X.

πgα-closed [1] If α-Cl(A) $\subset$ U whenever A $\subset$ U and U is π-open in X.

The finite union of regular open sets is said to be **π-open**. The complement of π-open set is said to be **π-closed** set. The complement of regular closed (resp. g-closed, π-open, πg-closed, α-closed, αg-closed, πgα-closed) set is said to be **regular open** (resp. **g-open**, **π-open**, **πg-open**, **α-open**, **αg-open** , **πgα-open**) sets. The intersection of all g-closed sets containing A is called the **g-closure of A** [**3**] and denoted by **Cl*(A)**, and the **g-interior of A** [**9**] is the unionof all g-open sets contained in A and is denoted by **Int*(A).**

Definition

A subset A of a topological space X is called,

Dα-closed [11] If Cl*(Int(Cl*(A))) ⊆ A .

Dαg-closed If $Cl_\alpha^D(A) \subseteq$ Uwhenever A ⊆ U, and U is open in X.

πgDα-closed If $Cl_\alpha^D(A) \subset$ U whenever A ⊂ U and U is π-open in X.

The complement of Dα closed (resp. Dαg-closed, πgDα-closed) sets is said to be **Dα-open** (resp. **Dαg-open**, **πgDα-open**).The intersection of all Dα-closed subsets of X containing A (i.e. super sets of A) is called the **Dα-closure of A** and is denoted by$\mathbf{Cl_\alpha^D(A)}$. The union of all Dα-open sets contained in A is called **Dα-interior of A** and is denoted by $\mathbf{Int_\alpha^D}$**(A)**.The family of all Dα-open (resp. Dα-closed) sets of a space X is denoted by **DαO(X)** (resp. **DαC(X)**).

Theorem [**11**].

Let X be a topological space. Then

1. Every α-closed subset of X is Dα-closed.
2. Every g-open subset of X is Dα-open.

We have the following implications for the properties of subsets.

closed	⇒	g-close	⇒	πg-closed
⇓		⇓		⇓
α-closed	⇒	αg-closed	⇒	πgα-closed
⇓		⇓		⇓
Dα-closed	⇒	Dαg-closed	⇒	πgDα-closed

Where none of the implications is reversible as can be seen from the following examples

Example

Let X = {a, b, c , d } and τ = { ϕ, {a}, {c, d}, {a, c, d }, {d}, {a, d}, X }. Then the set A = {a} is πgα-closed set as well πgDα-closed set but not g-closed set in X .

Example

Let X = {a, b, c , d } and τ = { ϕ, {a}, {c}, {a, b}, {a, c}, {a, d}, {a, d, c }, {a, b, d}, {a, b, c}, X}. Then the set A = {a, b} is πgα-closed set as well as πgDα-closed set but not αg-closed and not Dαg-closed set in X.Since A ⊂ {a, b, c} which is open by $Cl_\alpha^D \not\subset$ {a, b, c}.

Example

Let X = {a, b, c, d } and τ = { ϕ, {a}, {c, d}, {a, c, d}, {d}, {a, d}, X}. Then the set A = {c} is πgα-closed set as well as πgDα-closed set but not πg-closed set in X.

Theorem

1. Finite union of πgDα-closed sets are πgDα-closed.
2. Finite intersection of πgDα-closed need not be a πgDα-closed.
3. A countable union of πgDα-closed sets need not be a πgDα-closed.

Proof

1. Let A and B be πgDα-closed sets. Therefore $Cl_\alpha^D(A) \subset$ U and $Cl_\alpha^D(B) \subset$ U whenever A ⊂ U, B ⊂ U and U is π-open. Let A ∪ B ⊂ U where U is π-open. Since $Cl_\alpha^D(A \cup B) \subset Cl_\alpha^D(A) \cup Cl_\alpha^D(B) \subset$ U, we have A ∪ B is πgDα-closed.
2. Let X = {a, b, c, d} and τ = { ϕ, {a}, {b}, {a, b}, {a, b, c}, {a, b, d}, X}. Let A = {a, b, c}, B = {a, b, d}. A and B are πgDα-closed sets. But A ∩ B = {a, b}⊂{a, b} which is π-open. $Cl_\alpha^D(A \cap B) \not\subset$ {a, b}. Hence A ∩ B is not πgDα-closed.
3. Let R be the real line with the usual topology. Every singleton is πgDα-closed. But, A = {1/i : i = 2, 3, 4 ……………} is not πgDα-closed. Since A ⊂ (0, 1) which is π-open but $Cl_\alpha^D(A) \not\subset$ (0, 1).

Theorem

If A is πgDα-closed and A ⊂ B ⊂ $Cl_\alpha^D(A)$ then B is πgDα-closed.

Proof

Since A is πgDα-closed, $Cl_\alpha^D(A) \subset$ U whenever A ⊂ U and U is π-open. Let B ⊂ U and U be π-open. Since B ⊂ $Cl_\alpha^D(A)$, $Cl_\alpha^D(B) \subset Cl_\alpha^D(A) \subset$ U. Hence B is πgDα-closed.

Theorem

Let A be a πgDα-closed set in X. Then $Cl_\alpha^D(A)$ − A does not contain any non empty π-closed set.

Proof

Let F be a non empty π-closed set such that F ⊂$Cl_\alpha^D(A)$ − A. Then F ⊂ $Cl_\alpha^D(A)$ ∩ (X − A) ⊂ X − A implies A ⊂ X − F where X − F is π-open. Therefore $Cl_\alpha^D(A) \subset$ X − F implies F ⊂ $(Cl_\alpha^D(A))^C$. Now F ⊂ $Cl_\alpha^D(A) \cap (Cl_\alpha^D(A))^C$ implies F is empty.

Reverse implication does not hold.

Corollary

Let A be πgDα-closed. A is Dα-closed iff $Cl_\alpha^D(A)$ − A is π-closed.

Proof. Let A be Dα-closed set then A = $Cl_\alpha^D(A)$ implies $Cl_\alpha^D(A)$ − A = ϕ which is π-closed.

Conversely if $Cl_\alpha^D(A)$ − A is π-closed then A is Dα-closed.

Theorem

If A is π-open and πgDα-closed. Then A is Dα-closed hence clopen.

Proof

Let A be regular open. Since A is πgDα-closed, $Cl_\alpha^D(A) \subset$ A implies A is Dα-closed. Hence A is closed (Since every π-open, Dα-closed set is closed). Therefore A is clopen.

Theorem

For a topological space X, the following are equivalent :

1. X is extremally disconnected.
2. Every subset of X is πgDα-closed.
3. The topology on X generated by πgDα-closed sets.

Proof

(a) ⇒ (b). Assume X is extremally disconnected. Let A ⊂ U, where U is π-open in X.

Since U is π-open , it is the finite union of regular open sets and X is extremally disconnected, U is finite union of clopen sets and hence U is clopen. Therefore Cl_{α}^{D}(A) ⊂ Cl(A) ⊂ Cl(U) ⊂ U implies A is πgDα-closed.

(b) ⇒ (a). Let A be reguler open set of X. Since A is πgDα-closed by **Theorem 2.11** A is clopen. Hence X is extremally disconnected.

(b) ⇔ (c) is obvious.

Lemma[11]

If A is a subset of X, then

1. X – Cl_{α}^{D}(A) = Int_{α}^{D}(X – A).
2. Cl_{α}^{D}(X – A) = X – Int_{α}^{D}(A).

Theorem

A subset A of a topological space X is πgDα-open if F ⊂ Int_{α}^{D}(A) whenever F is π-closed and F ⊂ A.

Proof

Let F be π-closed set such that F ⊂ A. Since X – A is πgDα-closed and X – A ⊂ X – F we have F ⊂ Int_{α}^{D}(A).

Conversely, Let F ⊂ Int_{α}^{D}(A) where F is π-closed and F ⊂ A. Since F ⊂ A and X – F is π-open, Cl_{α}^{D}(X – A) = X – Int_{α}^{D}(A) ⊂ X – F. Therefore A is πgDα-open.

Theorem

If, Int_{α}^{D}(A) ⊂ B ⊂ A and A is πgDα-open then B is πgDα-open.

Proof

Since, Int_{α}^{D}(A) ⊂ B ⊂ A using **Theorem 2.8**, Cl_{α}^{D}(X – A) ⊃ (X – B) implies B is πgDα-open.

Remark

For any A ⊂ X, Int_{α}^{D}(Cl_{α}^{D}(A)) – A)) = ϕ.

Theorem

If A ⊂ X is πgDα-closed then Cl_{α}^{D}(A) – A is πgDα-open.

Proof

Let A be πgDα-closed and F be a π-closed set such that F ⊂ Cl_{α}^{D}(A) – A. By **Theorem 2.9**

F = ϕ implies F ⊂ Int_{α}^{D}(Cl_{α}^{D}(A) – A)). By **Theorem 2.14**, Cl_{α}^{D}(A) – A is πgDα-open.

Converse of the above theorem is not true.

Example

Let X = {a, b, c } and τ = { ϕ, {a}, {b}, {a, b}, X}. Let A = {b }.Then A is not πgDα-closed but Cl_{α}^{D}(A) – A = {a, b} πgDα-open.

Quasi Dα-normal spaces

Definition

A topological space X is said to be **Dα-normal** (resp. **quasi Dα-normal** , **mildly Dα-normal**) if for every pair of disjoint closed (resp. π-closed, regularly closed) subsets H, K of X, there exist disjoint Dα-open sets U, V of X such that H ⊂ U and K ⊂ V.

Example

Let X = {a, b, c, d} and τ = {ϕ, {a }, {b},{a, b}, {a, b, c }, X}. The pair of disjoint closed subsets of X are A = ϕ and B = {d}.Then Dα-closed sets in X are X, ϕ , {a}, {b}, {c}, {d}, {c, d}, {a, d}, {b, c},{a, c}, {b, d}, {a, c, d}, {b, c, d}. Also U = { b} and V = {c, d} are Dα-open sets such that A ⊂ U and B ⊂ V. Hence X is Dα-normal but it is not normal.

Example

Let X = { a, b, c, d } and τ = { ϕ, {a}, {c}, {a, c},{ b, d}, {a, b, d}, {b, c, d}, X }. The pair of disjoint π-closed subsets of X are A = {a} and B = {c}. Also U = {a} and V = {b, c, d} are open sets such that A ⊂ U and B ⊂ V. Hence X is quasi-normal as well as quasi Dα-normal because every open set is Dα-open set.

By the definitions and examples stated above, we have the following diagram:

normality ⇒	quasi-normality ⇒	mild-normality
⇓	⇓	⇓
Dα-normality ⇒	quasi Dα-normality ⇒	mild Dα-normality

Theorem

For topological space X , the following are equivalent:

a. X is quasi Dα-normal.
b. For any disjoint π-closed sets H and K, there exist disjoint Dαg-open sets U and V such that H⊂ U and K ⊂ V.
c. For any disjoint π-closed sets H and K, there exist disjoint πgDα-open sets U and V such that H ⊂ U and K ⊂ V.
d. For any π -closed set H and any π-open set V containing H, there exist a Dαg-open set U of X such that H ⊂ U ⊂ Cl_{α}^{D}(U) ⊂ V.
e. For any π-closed set H and any π-open set V containing H, there exist a πgDα-open set U of X such that H ⊂ U ⊂ Cl_{α}^{D}(U) ⊂ V.

Proof

(a) ⇒ (b), (b) ⇒ (c), (d) ⇒ (e), (c) ⇒ (d) and (e) ⇒ (a). (a) ⇒ (b).

Let X be quasi Dα-normal. Let H, K be disjoint π-closed sets of X. By assumption, there exist disjoint Dα-open sets U, V such that H ⊂ U and K ⊂ V. Since every Dα-open set is Dαg-open, U,V are Dαg-open sets such that H ⊂ U and K ⊂ V.

(b) ⇒ (c). Let H, K be two disjoint π -closed sets. By assumption, there exists Dαg-open sets U and V such that H ⊂ U and K ⊂ V. Since Dαg-open set is πgDα-open, U and V are πgDα-open sets such that H ⊂ U and K⊂ V.

(d) ⇒ (e). Let H be any π -closed set and V be any π -open set containing H. By assumption, there exist Dαg-open set U of X such that H ⊂ U ⊂ Cl_{α}^{D}(U) ⊂ V. Since every Dαg-open set is πgDα-open, there exist πgDα-open sets U of X such that H ⊂ U ⊂ Cl_{α}^{D}(U) ⊂ V.

(c) ⇒ (d). Let H be any π-closed set and V be any π-open set containing H. By assumption, there exist πgDα-open sets U and W such that H ⊂ U and X – V ⊂ W. By **Theorem 2.14**, we get X – V ⊂ Int_{α}^{D}(W) and Cl_{α}^{D}(U) ∩ Int_{α}^{D}(W) = ϕ. Hence H ⊂ U ⊂ Cl_{α}^{D}(U) ⊂ X – Int_{α}^{D}(W) ⊂ V.

(e) ⇒ (a). Let H, K be any two disjoint π-closed set of X. Then H ⊂ X – K and X – K is π -open. By assumption, there exist πg Dα-open set G of X such that $H \subset G \subset Cl_\alpha^D(G) \subset X - K$. Put $U = Int_\alpha^D(G)$, $V = X - Cl_\alpha^D(G)$. Then U and V are disjoint Dα-open sets of X such that $H \subset U$ and $K \subset V$.

Some Functions

Definition

A function $f : X \to Y$ is said to be

1. **Almost closed [10]**(resp. almost Dα-closed , **almost Dαg-closed)** if f (F) is closed **(resp. Dα-closed , Dαg-closed)** in Y for every $F \in RC(X)$.
2. **πgDα-closed (resp. almost πgDα-closed)** if for every closed set (resp. regularly closed) F of X , f(F) is πgDα-closed in Y.
3. **Π-continuous [2] (resp. πgα-continuous [1],** πgDα-continuous) if $f^{-1}(F)$ is π-closed (resp.πgα-closed, πgDα-closed) in X for every closed set F of Y.
4. **Almost continuous [10] (resp. almost π-continuous [2], almost πgα- continuous [1**], almost πgDα-continuous) if $f^{-1}(F)$ is closed (resp. π- closed, πgα-closed, πgDα-closed) in X for every regularly closed set F of Y.
5. Rc-preserving [6] if f(F) is regularly closed in Y for every $F \in RC(X)$.

From the definitions stated above, we obtain the following diagram:

closed ⇒ α-closed ⇒ αg-closed ⇒ πgα-closed
⇓ ⇓ ⇓ ⇓
al.-closed ⇒ al.Dα-closed ⇒ al. Dαg-closed ⇒al. πgDα-closed

Where al. = almost

Moreover, by the following examples, we realize that none of the implications is reversible.

Example

$X = \{a, b, c, d\}$, $\tau = \{\phi, X, \{c\}, \{a, b, d\}$ and $\sigma = \{\phi, \{a\}, \{c, d\}, \{a, c, d\} \{d\}, \{a, d\}, X\}$. Let $f : (X, \tau) \to (X, \sigma)$ be the identity function, then f is πgα-closed as well as πgDα-closed but not πg-closed. Since A= {c} is not πg-closed in (X, σ).

Example

Let $X = \{a, b, c, d\}$, $\tau = \{\phi , X, \{c\},\{a, b, d\}, \{b, d\}, \{b, c, d\}, X \}$ and $\sigma = \{\phi, X, \{a\},\{c, d\}, \{a, c, d\}, \{d\}, \{a, d\}\}$.Let $f : (X, \tau) \to (X, \sigma)$ be the identity function. Then f is almost πgα-closed as well as almost πgDα-closed but not πgDα-closed. Since A = {a} is not πgDα-closed

Theorem

If $f: X \to Y$ is an almost π-continuous and πgDα-closed function, then f(A) is πgDα-closed in Y for every πgDα-closed set A of X.

Proof

Let A be any πgDα-closed set A of X and V be any π-open set of Y containing f(A). Since f is almost π-continuous, $f^{-1}(V)$ is π-open in X and $A \subset f^{-1}(V)$. Therefore $Cl_\alpha^D(A) \subset f^{-1}(V)$ and hence $f(Cl_\alpha^D(A)) \subset V$. Since f is πgDα-closed, $f(Cl_\alpha^D(A))$ is πgDα-closed in Y. And hence we obtain $Cl_\alpha^D(f(A)) \subset Cl_\alpha^D(f(Cl_\alpha^D(A))) \subset V$. Hence f(A) is πgDα-closed in Y.

Theorem

A surjection $f : X \to Y$ is almost πgDα-closed if and only if for each subset S of Y and each $U \in RO(X)$ containing $f^{-1}(S)$ there exists a πgDα-open set V of Y such that $S \subset V$ and $f^{-1}(V) \subset U$.

Proof

Necessity, suppose that f is almost πgDα-closed. Let S be a subset of Y and $U \in RO(X)$ containing $f^{-1}(S)$. If $V = Y - f(X - U)$, then V is a πgDα-open set of Y such that $S \subset V$ and $f^{-1}(V) \subset U$.

Sufficiency, Let F be any regular closed set of X. Then $f^{-1}(Y - f(F)) \subset X - F$ and $X - F \in RO(X)$. There exists πgDα-open set V of Y such that $Y - f(F) \subset V$ and $f^{-1}(V) \subset X - F$. Therefore, we have $f(F) \supset Y - V$ and $F \subset X - f^{-1}(V) \subset f^{-1}(Y - V)$. Hence we obtain $f(F) = Y - V$ and f(F) is πgDα-closed in Y which shows that f is almost πgDα-closed.

Preservation Theorem

Theorem

If $f : X \to Y$ is an almost πgDα-continuous, rc-preserving injection and Y is quasi Dα-normal then X is quasi Dα-normal.

Proof

Let A and B be any disjoint π-closed sets of X. Since f is an rc-preserving injection, f(A) and f(B) are disjoint π-closed sets of Y. Since Y is quasi Dα-normal, there exist disjoint Dα-open sets U and V of Y such that $f(A) \subset U$ and $f(B) \subset V$.

Now if G = Int(Cl(U)) and H = Int(Cl(V)). Then G and H are regularly open sets such that $f(A) \subset G$ and $f(B) \subset H$. Since f is almost πgDα-continuous, $f^{-1}(G)$ and $f^{-1}(H)$ are disjoint πgDα-open sets containing A and B which shows that X is quasi Dα-normal.

Theorem

If $f: X \to Y$ is π-continuous, almost Dα-closed surjection and X is quasi Dα-normal space then Y is Dα-normal.

Proof

Let A and B be any two disjoint closed sets of Y. Then $f^{-1}(A)$ and $f^{-1}(B)$ are disjoint π-closed sets of X. Since X is quasi Dα-normal, there exist disjoint Dα-open sets of U and V such that $f^{-1}(A) \subset U$ and $f^{-1}(B) \subset V$. Let G = Int(Cl(V)) and H = Int(Cl(V)). Then G and H are disjoint regularly open sets of X such that $f^{-1}(A) \subset G$ and $f^{-1}(B) \subset H$. Set $K = Y - f(X - G)$ and $L = Y - f(X - H)$.Then K and L are Dα-open sets of Y such that $A \subset K$, $B \subset L$, $f^{-1}(K) \subset G$, $f^{-1}(L) \subset H$. Since G and H are disjoint, K and L are disjoint. Since K and L are Dα-open and we obtain $A \subset Int_\alpha^D(K)$,$B \subset Int_\alpha^D(L)$ and $Int_\alpha^D(K) \cap Int_\alpha^D(L) = \phi$.Therefore Y is Dα-normal.

Theorem

Let $f : X \to Y$ be an almost π-continuous and almost πgDα-closed surjection. If X is quasi Dα-normal space then Y is quasi Dα-normal.

Proof

Let A and B be any disjoint π-closed sets of Y. Since f is almost π-continuous, $f^{-1}(A)$, $f^{-1}(B)$ are disjoint closed subsets of X. Since X is quasi Dα-normal, there exist disjoint Dα-open sets U and V of X such that $f^{-1}(A) \subset U$ and $f^{-1}(B) \subset V$.

Let G = Int(Cl(U)) and H = Int(Cl(V)). Then G and H are disjoint regularly open sets of X such that $f^{-1}(A) \subset G$ and $f^{-1}(B) \subset H$. By **Theorem 4.5**, there exist πgDα-open sets K and L of Y such that $A \subset K$, $B \subset L$, $f^{-1}(K) \subset G$ and $f^{-1}(L) \subset H$. Since G and H are disjoint, so are K and L by **Theorem 2.14**, $A \subset \mathrm{Int}_\alpha^D(K)$, $B \subset \mathrm{Int}_\alpha^D(L)$ and $\mathrm{Int}_\alpha^D(K) \cap \mathrm{Int}_\alpha^D(L) = \phi$. Therefore Y is quasi Dα-normal.

Corollary

If $f : X \to Y$ is almost continuous and almost closed surjection and X is a normal space, then Y is quasi Dα-normal.

Proof

Since every almost closed function is almost πgDα-closed so Y is quasi Dα-normal.

Conclusion

The notion of quasi Dα-normal in topological spaces has been generalized and obtain characterizations and preservation theorems of quasi Dα-normal.

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